Hybrid Rumor Debunking in Online Social Networks: A Differential Game Approach

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***Abstract*—Online social networks (OSNs) facilitate the rapid and extensive spreading of rumors. While most existing methods for debunking rumors consider a solitary debunker, they overlook that rumor-mongering and debunking are interdependent and confrontational behaviors. In reality, a debunker must consider the impact of rumor-mongering behavior when making decisions. Moreover, a single rumor-debunking strategy is ineffective in addressing the complexity of the rumor environment in networks. Therefore, this paper proposes a hybrid rumor-debunking approach that combines truth dissemination and regulatory measures based on the differential game theory under adversarial behaviors of rumor-mongering and debunking. Towards this end, we first establish a rumor propagation model using node-based modeling techniques that can be applied to any network structure. Next, we mathematically describe and analyze the processes of rumor-mongering and debunking. Finally, we validate the theoretical results of the proposed method through various comparative experiments, including comparisons with a random strategy, a uniform strategy, and single strategy models on real-world datasets collected from Facebook, Twitter, and YouTube. Furthermore, we harness two actual rumor events to estimate parameters and predict rumor propagation, thereby affirming the veracity and effectiveness of our rumor propagation model.**

***Index Terms*—Online social network, rumor propagation, differential game, hybrid debunking**

I. INTRODUCTION

WITH the development of communication technology, the Internet connects people or organizations with a series of social relationships, forming Online Social Networks (OSNs) [1]. OSNs have become the primary platform for information acquisition and dissemination, offering various real-time information services and easy communication that has penetrated almost every aspect of daily life [2]. As a result, OSNs have garnered significant attention from both industry and academia, specifically in the areas of information dissemination [3] and public opinion monitoring [4].

Regrettably, the inherent openness and collaborative nature of OSNs have facilitated the proliferation of rumors, malicious speech, and false information [5]. Within a short period of time, rumors can diffuse widely through OSNs, leading to significant economic repercussions [6], societal unrest [7], and triggering a series of events that profoundly influence public opinion. Clearly, rumors in OSNs constitute a significant menace to both cybersecurity and social stability. Consequently, an urgent need exists to analyze the process of rumor propagation in OSNs and devise effective strategies for debunking them.

The dissemination of rumors within a network is a complex process, which due to the numerous factors involved is inherently difficult to model. Nevertheless, several models have been designed to simulate the evolutionary dynamics of rumor dissemination. The majority of existing models are rooted in epidemiology [8], a discipline that classifies populations into distinct states and then analyzes the dynamics of disease dissemination, a process that, conceptually, is very similar to rumor propagation. Building upon this foundation, researchers have further devised novel models that integrate social network structures with user attributes to depict the process of rumor dissemination in OSNs [9], [10], [11]. Nonetheless, the aforementioned studies primarily focused on modeling OSNs utilizing homogeneous mixed network or scale-free network models. These studies made assumptions in regard to the degree of distribution of network nodes, approximating it with either a Poisson or power-law distribution.

In reality, OSNs display intricate structures wherein every user functions as both a sender and receiver of information, with individual interactions and the network environment collectively influencing the dissemination of rumors. Node-based modeling approaches [12], [13] facilitate individualized user modeling by utilizing differential dynamical systems to characterize the probabilistic evolution of users across different states. This approach proficiently describes the processes of dissemination on various networks. Consequently, the establishment of dynamical models capable of adapting to various network structures emerged as a pivotal task for delineating the intricate dynamics of rumor evolution within OSNs. Faced with rumors in OSNs, it is necessary to take measures to minimize their impact. Two main approaches are commonly

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employed to suppress the spread of rumors. One approach involves blocking the spread of rumors [7], [14], while the other focuses on publishing the truth to clarify the rumors [15], [16]. However, a single method alone cannot effectively address the complexities of the current rumor environment in online networks, which include situations involving extremism such as terrorist attacks, malicious defamation, or incitement of hate speech. Therefore, researchers in the field have explored the coordinated implementation of multiple strategies to mitigate the impact of rumors [17], [18].

Prior literature [19], [20] proposed hybrid strategies that combine both truth propagation and blocking methods in order to effectively control rumors in networks. However, most of the existing works on hybrid debunking strategies largely focus on the debunking side, overlooking the confrontations and interactions between rumor-mongering and debunking behaviors. This oversight weakens the accuracy and effectiveness of hybrid debunking strategies. Thus, there is a need to study hybrid strategies that comprehensively consider the interaction between rumor-mongering and debunking behaviors.

Motivated by the above discussion, this paper investigates the issue of hybrid debunking strategies in the face of adversarial behaviors between rumor-mongering and debunking, employing node-based modeling techniques and differential game methods. Node-based dynamical models are utilized in our work due to their ability to effectively describe the process of rumor propagation in networks with arbitrary structures, while accurately estimating the resultant losses. Similarly, the differential game theory is used because of its usefulness for the analysis of the adversarial behaviors and decision-making techniques of participants over continuous time, thus, enabling the discovery of effective debunking strategies. Using the outlined methodology, we make the following contributions in this paper:

1) We present a novel node-based dynamical model for analyzing the propagation of rumors in OSNs that is suitable for diverse network structures. The model's dynamic evolution captures the influence of competitive interactions between rumor and truth propagation, alongside the involvement of regulatory authorities.
2) We employ differential game theory to investigate the dynamics of rumor-mongering and debunking behaviors, and present a hybrid debunking strategy that integrates truth dissemination and regulatory measures.
3) We derive an optimality system to determine the Nash equilibrium, and design an algorithm that provides numerical solutions for achieving said equilibrium. Through comparisons with random and uniform strategies, as well as models solely focused on single strategies, we validate the efficacy of the proposed method using multiple real datasets and two actual rumor events.

II. Related work

A considerable amount of work has been done on the topic of rumor debunking over recent years [21], [22]. While numerous techniques have been proposed in the literature, the collaborative use of various debunking strategies has been found to be among the most effectively solutions to suppress the spread of rumors. Xiong *et al.* [23], for example, proposed and systematically studied multiple methods to inhibit information diffusion from an epidemiological perspective, assessing the differences and combined effects of these methods. Wen *et al.* [24] investigated and confirmed the superior inhibitory effects of two cooperative strategies compared to the consideration of a single strategy on OSNs. Furthermore, Yang *et al.* [25] developed a competition propagation model between rumors and truths, and assessed the effectiveness of hybrid debunking strategies. These studies demonstrate the efficacy of collaborative efforts involving different strategies in mitigating the impact of rumors, yet they disregarded the associated implementation costs. Both those propagating rumors and those debunking them are typically limited by resources and costs. Consequently, rational selection of debunking strategies is essential for resource allocation efficiency.

Considering cost constraints, Lin *et al.* [26] proposed an information diffusion model based on a homogeneous mixed network. Specifically, the authors developed two collaborative control strategies to minimize losses resulting from the spread of fraudulent information and determined the optimal distribution of these strategies. Huang *et al.* [27] proposed a false-information propagation model with a sequential clarification mechanism, and framed the problem as a three-layer optimization task to suppress the propagation of false information effectively. Yao *et al.* [18] introduced the multi-probability independent cascade (MPIC) model, wherein different control measures were implemented based on users' susceptibility to rumors. This approach facilitated cost-effective rumor containment. Cheng *et al.* [11] constructed a dual-layer model that captures the interplay between rumor propagation and social media. Here, the authors integrated post-deletion, popularization education, and immune treatment as diverse strategies to mitigate the extent of rumor propagation while minimizing associated costs. Chai *et al.* [17] introduced the node-based susceptible-infected-recovered-susceptible (SIRS) model and presented two collaborative implementation strategies: one aimed at suppressing the spread of negative information, while the other aimed to enhance the dissemination of positive information. Furthermore, Ding *et al.* [20] developed a rumor model based on a scale-free network and proposed a hybrid strategy combining the pulse-blocking of rumors with the continuous dissemination of truths to effectively suppress the spread of rumors. The aforementioned studies have shown the cost-efficiency of employing multiple collaborative strategies, but predominantly focused on the actions of rumor-debunkers, while disregarding the adversarial and interdependence of rumor-mongering and debunking behaviors.

Furthermore, it is important to highlight that the process of rumor dissemination is often accompanied by rumor-debunking actions, the dynamics of rumor evolution are also significantly influenced by this adversarial interplay. Game theory offers theoretical tools for analyzing such adversarial (decision-making) problems. Chu *et al.* [28] utilized the differential game theory to model the confrontation between cyberbullying and anti-cyberbullying and mitigate the negative effects of cyberbullying in a cost-effective manner. Wan *et* *al.* [29], for instance, examined the coexistence and conflicts among multiple pieces of information within OSNs. The authors investigated the spread of positive and negative information using evolutionary game theory, allowing for the optimal allocation of control resources. From a multi-dimensional standpoint, Xiao *et al.* [30] introduced a rumor propagation model grounded in evolutionary game theory and a data augmentation mechanism. Their model considered various types of information, encompassing both rumors and anti-rumors. Mou *et al.* [31] developed a rumor propagation model that considers various kinds of information, including rumors, anti-rumors, and motivation rumors, based on evolutionary game theory. While these studies examined the adversarial relationship among different types of information, they did not specifically address the adversarial and interactive dynamics between rumor-mongering and debunking behaviors. While Huang *et al.* [32] did consider the adversarial behaviors exhibited by both rumor-mongers and debunkers and suggested propagating truths to mitigate the impact of rumors, their proposed approach focused solely on a single strategy, making it suboptimal from an effectiveness perspective.

In conclusion, this paper introduces a node-based dynamical model that is applicable to various network structures. It describes the processes of rumor-mongering and debunking using differential game theory and presents an optimal hybrid debunking strategy for the collaborative implementation of two methods. In contrast to prior research, this paper presents a hybrid debunking strategy that considers the adversarial behaviors of both rumor-mongering and debunking.

III. Evolutionary dynamical model

In this section, we first describe the background on rumor-mongering and debunking in OSNs, and then introduce a corresponding evolutionary dynamical model for the analysis.

A. Background

The process of rumor and rumor-debunking propagation in an OSN is presented in Fig. 1. On the one hand, *the rumor-monger disseminates rumors* and continuously pushes new supporting information to other users. Some recipients of the rumor further propagate it to their friends, leading to continuous spread. On the other hand, as *the truth is published by the rumor-victim to debunk the rumors*, the network regulatory authority takes measures to control the spread of rumors. The rumor-victim and the network regulatory authority collectively constitute the rumor-debunker. When confronted with rumors, users tend to exhibit one of three behaviors/attitudes: (1) they are compelled to believe the rumor due to its persuasiveness (believable), (2) they firmly believe in the truth and are dismissing the rumor (refuted), or (3) they maintain a neutral stance, neither believing the rumor nor the truth (doubtful). Given the implementation of regulatory measures in reality that hinder rumor dissemination, there also exists a fourth type of behavior, where users are inclined to believe the rumor but are restrained from spreading it. Over time, the behaviors and attitude of users evolve under the influence of online friends and confrontations between the rumor-monger and debunker.

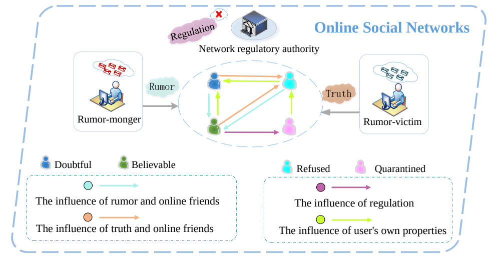


Fig. 1. The rumor and rumor-debunking propagation process.

Throughout the rumor-propagation process, both parties allocate resources to support their activities (driven by their own interests), resulting in a complex two-sided game. Our goal in this paper is to find cost-effective hybrid rumor-debunking strategies for the outlined scenario by combining truth dissemination and regulatory measures under the adversarial behaviors between rumor-mongering and debunking.

B. Dynamical model formulation

Let an undirected graph $G = \{U, E\}$ represent the network structure of OSNs, where U and E denote the nodes and edges. Here, $U = \{u_1, u_2, \cdots, u_N\}$ and E represent the set of online users and the information-interaction relationships between the users, respectively, and N is the number of network nodes. The corresponding adjacency matrix of G is denoted as $\mathbf{A} = (a_{ij})_{N\times N}$, $a_{ij} = 1$ if $(u_i, u_j) \in E$, and $a_{ij} = 0$ otherwise.

Considering the different behaviors and user attitudes, described in the previous section, each user in an OSN can only be in one of four states: (1) *Believable* (B) denotes that the online user believes in this rumor and disseminates it, (2) *Refuted* (R) denotes that the online user does not believe in this rumor and propagates the truth, (3) *Doubtful* (D) denotes that the online user neither believes the rumor nor believes the truth, and (4) *Quarantined* (Q) denotes a user that believed the rumor (i.e., was in the B state) but was blocked (e.g., due to regulatory measures) and can therefore not spread it further.

We represent the state of user u_i at time instance $t \in [0, T]$ as $X_i(t)$, and the vector $\mathbf{X}(t) = (X_1(t), X_2(t), \ldots, X_N(t))$ to denote the state of the OSN at time t. Furthermore, we use $X_i(t) = 0$, $X_i(t) = 1$, $X_i(t) = 2$ and $X_i(t) = 3$ to encode that at time instance t, the user u_i is in the D, B, R and Q states, respectively. Finally, if we denote the probabilities (Pr) that u_i at time instance t exists in one of the four states as $D_i(t)$, $B_i(t)$, $R_i(t)$, $Q_i(t)$ existing in the four states at t, then the following relations can be derived:

$$D_i(t) + B_i(t) + R_i(t) + Q_i(t) = 1. \tag{1}$$

$$\begin{aligned} D_i(t) &= \Pr\{X_i(t) = 0\}, B_i(t) = \Pr\{X_i(t) = 1\}, \\ R_i(t) &= \Pr\{X_i(t) = 2\}, Q_i(t) = \Pr\{X_i(t) = 3\}. \end{aligned} \tag{2}$$

Due to the influence of online friends and the overall network environment, the state of users evolves over time. To capture the evolution process, we model multiple aspects of the rumor propagation and debunking processes using various probabilities. The probabilities capture various assumptions, in which the state of nodes will influence each other and change jointly. Specifically, we introduce the following probabilities/variables into our model:

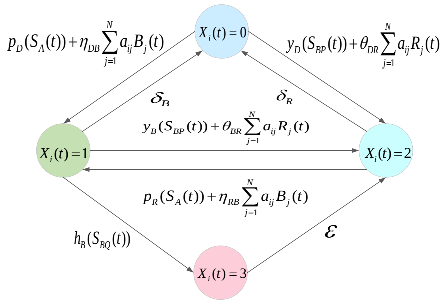


Fig. 2. The developed state transfer diagram.

1) η_{DB}/η_{RB}: The probability of a rumor-doubtful/rumor-refuting user turning into a user believing the rumor due to the influence of an OSN friend who believes the rumor.
2) θ_{DR}/θ_{BR}: The probability of a rumor-doubtful/rumor-believing user turning into a refuting user that does not believe the rumor due to the influence of an OSN friend who also does not believe in the rumor.
3) $p_D(S_A(t))/p_R(S_A(t))$: The probability of a rumor-doubtful/rumor-refuting user turning into a believable one due to the influence of rumor-mongering information.
4) $y_D(S_{BP}(t))/y_B(S_{BP}(t))$: The probability of a rumor-doubtful/rumor-believing user turning into a user that does not believe the rumor due to the influence of rumor-debunking information.
5) $h_B(S_{BQ}(t))$: The probability of a rumor-believing user turning into a quarantined user due to the influence of network regulatory measures.
6) δ_B/δ_R: The probability of a rumor-believing/rumor-refusing user turning into a rumor-doubtful user due to the influence of fading memory and diminishing interest.
7) ε: The probability of a quarantined user turning into a rumor-refuting user due to the change in attitude towards the rumor.

Based on the probabilities introduced above, we define a state transfer diagram, as shown in Figure. 2.

According to the Kolmogorov forward equation for Markov chains [33], the model can be represented by the system in Eq. (3) with the initial condition $D_i(0), B_i(0), R_i(0) \in [0,1]$, $t \in [0,T]$, where $[0,T]$ denotes the entire time range for the rumor-mongering and debunking process. Due to the $Q_i(t) = 1 - D_i(t) - B_i(t) - R_i(t)$, the expected state of the OSN at time t can be represented as $\mathbf{E}(t) = (D_1(t), \ldots, D_N(t), B_1(t), \ldots, B_N(t), R_1(t), \ldots, R_N(t))$, where $\mathbf{E}_0 = \mathbf{E}(0)$ represents the initial state of the network.

IV. Rumor-mongering and Rumor-debunking Differential Game Problem

Due to the adversarial behaviors between rumor-mongering and debunking, this problem can be characterized as a two-sided game and analyzed using differential game theory. In this section, we therefore, first (1) mathematically formalize the rumor-mongering and rumor-debunking strategies; (2) quantify the expected net gain for the rumor-monger and the total expected loss for the rumor-debunker; and finally (3) formulate the studied problem within a game-theory framework.

A. Rumor-mongering strategy and rumor-debunking strategy

Let us denote the rumor-monger and rumor-debunker as A and B, respectively. For A, the cost of publishing rumor-mongering information within $[0,t]$ is represented by $C_A(t)$. On this basis, we refer to $S_A(t) = \frac{dC_A(t)}{dt}$ as the rumor-mongering rate at time t. In reality, due to limited resources available to A, $S_A(t)$ is commonly bounded, and we denote the upper bound as $\overline{S}_A$. For ease in implementing the strategy, it is assumed that the rumor-mongering strategy S_A is piecewise continuous. Let $S_A \in PC[0,T]$, where $PC[0,T]$ denotes the set of all piecewise continuous functions defined on the interval $[0,T]$. Thus, the feasible set of rumor-mongering strategies can be represented as follows:

$$\mathbb{N}_A = \left\{ S_A \in PC[0,T] \,\middle|\, S_A(t) \le \overline{S}_A, 0 \le t \le T \right\}. \quad (4)$$

For B, let $C_{BP}(t)$ represent the cumulative cost of pushing truth within $[0,t]$. On this basis, we refer to $S_{BP}(t) = \frac{dC_{BP}(t)}{dt}$ as the truth dissemination rate at time t. Furthermore, let $C_{BQ}(t)$ represent the cost of regulatory measures within $[0,t]$, and let $S_{BQ}(t) = \frac{dC_{BQ}(t)}{dt}$ represent the regulatory rate at time t. Similarly as above, let $\overline{S}_B = (\overline{S}_{BP}, \overline{S}_{BQ})$ be the upper bound of the hybrid debunking strategy. Thus, the feasible set of rumor-debunking strategies can be represented as follows:

$$\mathbb{N}_B = \left\{ S_B \in PC\,[0,T] \,\middle|\, S_B(t) \le \overline{S}_B, 0 \le t \le T \right\}. \quad (5)$$

In the following sections, we explore the optimal strategy pairs for the rumour-monger and debunker within $\mathbb{N}_A$ and $\mathbb{N}_B$.

B. Rumor-mongering gain and rumor-debunking loss

In order to identify cost-effective hybrid rumor-debunking strategies, it is necessary to assess the rumor-monger's expected net gain and the rumor-debunker's expected total loss. Throughout the time interval $[0,T]$, let the functions $L_A(S_A, S_B)$ and $L_B(S_A, S_B)$ denote the rumor-mongering gain and rumor-debunking loss, respectively. Given a strategy

$$\begin{cases} \frac{dD_i(t)}{dt} = \delta_B B_i(t) + \delta_R R_i(t) - \left[p_D(S_A(t)) + \eta_{DB}\sum_{j=1}^{N} a_{ij}B_j(t) + y_D(S_{BP}(t)) + \theta_{DR}\sum_{j=1}^{N} a_{ij}R_j(t) \right] D_i(t), \\ \frac{dB_i(t)}{dt} = \left[p_D(S_A(t)) + \eta_{DB}\sum_{j=1}^{N} a_{ij}B_j(t) \right] D_i(t) - \left[\delta_B + y_B(S_{BP}(t)) + \theta_{BR}\sum_{j=1}^{N} a_{ij}R_j(t) + h_B(S_{BQ}(t)) \right] B_i(t) + \left[p_R(S_A(t)) + \eta_{RB}\sum_{j=1}^{N} a_{ij}B_j(t) \right] R_i(t), \\ \frac{dR_i(t)}{dt} = \left[y_D(S_{BP}(t)) + \theta_{DR}\sum_{j=1}^{N} a_{ij}R_j(t) - \varepsilon \right] D_i(t) - \left[\delta_R + p_R(S_A(t)) + \eta_{RB}\sum_{j=1}^{N} a_{ij}B_j(t) + \varepsilon \right] R_i(t) + \left[y_B(S_{BP}(t)) + \theta_{BR}\sum_{j=1}^{N} a_{ij}R_j(t) - \varepsilon \right] B_i(t) + \varepsilon, \\ 0 \le t \le T, 1 \le i \le N. \end{cases} \quad (3)$$

pair (S_A, S_B), the rumor-monger's net gain is the total gain from rumor-believing users minus the cost coming from the implementation of the rumor-mongering strategy S_A. While the rumor-debunker's expected total loss consists of the loss incurred by rumor-believing users and the cost coming from the implementation of the rumor-debunking strategy S_B. To capture these considerations, we define:

1) The gain (loss) that a rumor-believing user brings to $A(B)$ in a unit of time as $w_A(w_B)$.
2) The rate at which A diffuses rumor-mongering information as S_A, the rate at which B diffuses truth (implementing of regulatory measures) as $S_{BP}(S_{BQ})$.

According to these definitions, at the infinitesimal time interval $[t, t+dt]$, the gain accrued to A by the rumor-believing users is $\sum_{i=1}^{N} w_A B_i(t)dt$ and the cost of publishing rumor-mongering information is $S_A(t)dt$. Therefore, the expected net gain of the rumor-monger over the time interval $[0, T]$ equals

$$L_A(S_A, S_B) = \int_0^T \sum_{i=1}^{N} w_A B_i(t)dt - \int_0^T S_A(t)dt. \tag{6}$$

Similarly, at the infinitesimal time interval $[t, t+dt]$, the loss accrued to B by the rumor-believing users is $\sum_{i=1}^{N} w_B B_i(t)dt$, the cost of publishing truth is $S_{BP}(t)dt$ and the cost of implementing regulatory measures is $S_{BQ}(t)dt$. Hence, the expected total loss of the rumor-debunker over the time interval $[0, T]$ can be expressed as:

$$\begin{aligned} L_B(S_A, S_B) = \int_0^T \sum_{i=1}^{N} w_B B_i(t)dt + \int_0^T S_{BP}(t)dt + \\ \int_0^T S_{BQ}(t)dt. \end{aligned} \tag{7}$$

C. Differential game problem formulation

Based on the above discussion, one can see that the rumor-monger wants to maximize the gain as much as possible, while the rumor-debunker wants to minimize the loss as much as possible. This type of adversary behavior can be framed as a game theory problem and can be approached using the Nash equilibrium solution. Here, the Nash equilibrium refers to a strategy pair in a game where no player can improve their payoff by unilaterally changing their strategy, resulting in a situation where all players have achieved their maximum possible payoff, i.e., the equilibrium.

Throughout the time interval $t \in [0, T]$, the mathematical expression for the rumor-mongering and rumor-debunking differential game problem under the system in Eq. (3) can be expressed as follows:

$$\begin{aligned} &\underset{(S_A, S_B)\in(\mathbb{N}_A, \mathbb{N}_B)}{\text{Maximize}} L_A(S_A, S_B), \\ &\underset{(S_A, S_B)\in(\mathbb{N}_A, \mathbb{N}_B)}{\text{Minimize}} L_B(S_A, S_B). \end{aligned} \tag{8}$$

The goal of the game problem is to find the Nash equilibrium, that is, a strategy pair $(S_A^*, S_B^*) \in (\mathbb{N}_A, \mathbb{N}_B)$ that meets the following conditions:

$$\begin{aligned} L_A(S_A^*, S_B^*) \geq L_A(S_A, S_B^*), \forall S_A \in \mathbb{N}_A, \\ L_B(S_A^*, S_B^*) \leq L_B(S_A^*, S_B), \forall S_B \in \mathbb{N}_B. \end{aligned} \tag{9}$$

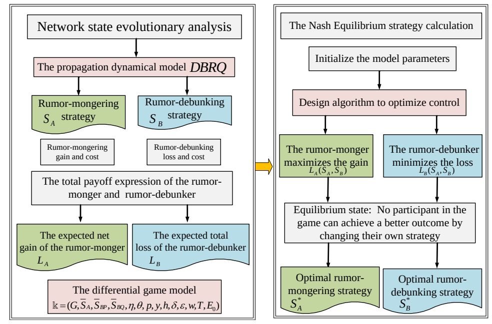


Fig. 3. The overall process flow of the proposed method.

Combining all introduced variables, we can see that the rumor-mongering and rumor-debunking differential game problem is determined by a 14-tuple of the following form:

$$\mathbb{k} = (G, \overline{S}_A, \overline{S}_{BP}, \overline{S}_{BQ}, \eta, \theta, p, y, h, \delta, \varepsilon, w, T, \mathbf{E}_0). \tag{10}$$

Assuming that the strategy pair (S_A^*, S_B^*) represents a Nash equilibrium for the rumor-mongering and rumor-debunking differential game problem, then it is fitting for the rumor-debunker to choose the rumor-debunking strategy S_B^* in any circumstance. On the one hand, if the rumor-debunker adheres to the rumor-debunking strategy S_B^*, then the rumor-monger must choose the rumor-mongering strategy S_A^* to maximize their gain. On the other hand, if the rumor-monger persists with the rumor-mongering strategy S_A^*, deviating from S_B^* will not lead to a reduction in costs for the rumor-debunker. Therefore, the strategy pair (S_A^*, S_B^*) is acceptable for both A and B. Next, our goal is to determine the Nash equilibrium for this problem.

V. Solving of Rumor-Mongering and Rumor-Debunking Differential Game Problem

In this section, we solve the rumor-mongering and rumor-debunking differential game problem. We first derive an optimality system for this problem and then design an algorithm to solve for the optimality system numerically. To have a more intuitive understanding of the rumor-mongering and rumor-debunking differential game problem and its solution, Fig. 3 illustrates the overall process of the proposed method.

A. Optimality System

To derive a system for determining the Nash equilibrium, it is necessary to establish the conditions for the Nash equilibrium. Towards this end, we first introduce two Hamiltonians of the rumor-monger H_A and rumor-debunker H_B in accordance with the differential game theory [34], i.e.:

$$\begin{aligned} H_A(\mathbf{E}, S_A, S_B, \lambda) = \sum_{i=1}^{N} w_A B_i - S_A + \sum_{i=1}^{N} \lambda_i^A \frac{dD_i}{dt} \\ + \sum_{i=1}^{N} \lambda_i^B \frac{dB_i}{dt} + \sum_{i=1}^{N} \lambda_i^C \frac{dR_i}{dt}, \end{aligned} \tag{11}$$

$$
\begin{aligned}
H_B(\mathbf{E}, S_A, S_B, \mu) = &\sum_{i=1}^{N} w_B B_i + S_{BP} + S_{BQ} + \sum_{i=1}^{N} \mu_i^A \frac{dD_i}{dt} \\
&+ \sum_{i=1}^{N} \mu_i^B \frac{dB_i}{dt} + \sum_{i=1}^{N} \mu_i^C \frac{dR_i}{dt},
\end{aligned} \tag{12}
$$

where $\lambda = \left(\lambda_1^A, \ldots, \lambda_N^A, \lambda_1^B, \ldots, \lambda_N^B, \lambda_1^C, \ldots, \lambda_N^C\right)$ and $\mu = \left(\mu_1^A, \ldots, \mu_N^A, \mu_1^B, \ldots, \mu_N^B, \mu_1^C, \ldots, \mu_N^C\right)$ are their respective adjoints.

Theorem 1. *Assume (S_A, S_B) is a Nash equilibrium for the rumor-mongering and rumor-debunking game problem and E is the solution to the model in (3). The following results can be obtained.*

1) There exist λ and μ, such that the model in (13) is valid, where $\lambda(T) = \mu(T) = 0$. 2) For $0 \le t \le T$, $1 \le i \le N$, let

$$
\begin{aligned}
Z_A^t(S) = &p_D(S) \sum_{i=1}^{N} \left[\lambda_i^B(t) - \lambda_i^A(t)\right] D_i(t) \\
&+ p_R(S) \sum_{i=1}^{N} \left[\lambda_i^B(t) - \lambda_i^C(t)\right] R_i(t) - S,
\end{aligned} \tag{14}
$$

There holds

$$
S_A(t) \in \arg \max_{S \in [0, \overline{S}_A]} Z_A^t(S). \tag{15}
$$

3) For $0 \le t \le T$, $1 \le i \le N$, let

$$
\begin{aligned}
Z_{BP}^t(S) = &y_D(S) \sum_{i=1}^{N} \left[\mu_i^C(t) - \mu_i^A(t)\right] D_i(t) \\
&+ y_B(S) \sum_{i=1}^{N} \left[\mu_i^C(t) - \mu_i^B(t)\right] B_i(t) + S,
\end{aligned} \tag{16}
$$

$$
Z_{BQ}^t(S) = S - h_B(S) \sum_{i=1}^{N} \mu_i^B(t) B_i(t), \tag{17}
$$

There holds

$$
S_{BP}(t) \in \arg \min_{S \in [0, \overline{S}_{BP}]} Z_{BP}^t(S), \tag{18}
$$

$$
S_{BQ}(t) \in \arg \min_{S \in [0, \overline{S}_{BQ}]} Z_{BQ}^t(S). \tag{19}
$$

Proof. Based on Pontryagin's maximum / minimum principle [34], there are λ and μ such that for $0 \le t \le T, 1 \le i \le N$, the following equation (20) holds.

$$
\begin{cases}
\dfrac{d\lambda_i^A(t)}{dt} = -\dfrac{\partial H_A(\mathbf{E}(t), S_A(t), S_B(t), \lambda(t))}{\partial D_i}, \\
\dfrac{d\lambda_i^B(t)}{dt} = -\dfrac{\partial H_A(\mathbf{E}(t), S_A(t), S_B(t), \lambda(t))}{\partial B_i}, \\
\dfrac{d\lambda_i^C(t)}{dt} = -\dfrac{\partial H_A(\mathbf{E}(t), S_A(t), S_B(t), \lambda(t))}{\partial R_i}, \\
\dfrac{d\mu_i^A(t)}{dt} = -\dfrac{\partial H_B(\mathbf{E}(t), S_A(t), S_B(t), \mu(t))}{\partial D_i}, \\
\dfrac{d\mu_i^B(t)}{dt} = -\dfrac{\partial H_B(\mathbf{E}(t), S_A(t), S_B(t), \mu(t))}{\partial B_i}, \\
\dfrac{d\mu_i^C(t)}{dt} = -\dfrac{\partial H_B(\mathbf{E}(t), S_A(t), S_B(t), \mu(t))}{\partial R_i}.
\end{cases} \tag{20}
$$

The system (13) can be obtained through direct computation. Since the terminal cost is unspecified, the final state remains unconstrained, $\lambda(T) = \mu(T) = 0$. Applying Pontryagin's maximum/minimum principle, we obtain

$$
\begin{aligned}
&S_A \in \arg \max_{\tilde{S} \in \mathbb{N}_A} H_A(\mathbf{E}, \tilde{S}, S_B, \lambda), \\
&S_B \in \arg \min_{\tilde{S} \in \mathbb{N}_B} H_A(\mathbf{E}, S_A, \tilde{S}, \mu),
\end{aligned} \tag{21}
$$

leading to Eqs. (15), (18) and (19), which completes the proof. □

The systems in (3), (13), (15), (18), (19), and the conditions $\lambda(T) = 0$ and $\mu(T) = 0$ constitute an optimality system of the rumor-mongering and rumor-debunking differential game problem. The optimality system can be solved using a numerical approach for finding the optimal strategy pair.

B. Numerical solution algorithm

In order to solve the optimality system, we design an algorithm (summarized in Algorithm 1) based on the forward-backward sweep method for solving ordinary differential equations and generating the Nash equilibrium [35].

The algorithm incorporates the forward-backward sweep and Euler's method and follows an iterative procedure. The process starts with an initial strategy combination that serves as a preliminary estimate of the solution, followed by iterative

$$
\begin{cases}
\dfrac{d\lambda_i^A(t)}{dt} = [\lambda_i^A(t) - \lambda_i^B(t)] \left[p_D(S_A(t)) + \eta_{DB} \sum_{j=1}^{N} a_{ij} B_j(t)\right] + [\lambda_i^A(t) - \lambda_i^C(t)] \left[y_D(S_{BP}(t)) + \theta_{DR} \sum_{j=1}^{N} a_{ij} R_j(t)\right] + \varepsilon \lambda_i^C(t), \\
\dfrac{d\lambda_i^B(t)}{dt} = -w_A - \delta_B \lambda_i^A(t) + [\delta_B + h_B(S_{BQ}(t))] \lambda_i^B(t) + \varepsilon \lambda_i^C(t) + M_1 \left[y_B(S_{BP}(t)) + \theta_{BR} \sum_{j=1}^{N} a_{ij} R_j(t)\right] + \eta_{RB} \sum_{j=1}^{N} a_{ij} R_j(t) M_2 + \eta_{DB} \sum_{j=1}^{N} a_{ij} D_j(t) M_3, \\
\dfrac{d\lambda_i^C(t)}{dt} = -\delta_R \lambda_i^A(t) + (\varepsilon + \delta_R) \lambda_i^C(t) - M_1 \left[p_R(S_A(t)) + \eta_{RB} \sum_{j=1}^{N} a_{ij} B_j(t)\right] + \theta_{DR} \sum_{j=1}^{N} a_{ij} D_j(t) [\lambda_j^A(t) - \lambda_j^C(t)] - \theta_{BR} \sum_{j=1}^{N} a_{ij} B_j(t) M_2, \\
\dfrac{d\mu_i^A(t)}{dt} = [\mu_i^A(t) - \mu_i^B(t)] \left[p_D(S_A(t)) + \eta_{DB} \sum_{j=1}^{N} a_{ij} B_j(t)\right] + [\mu_i^A(t) - \mu_i^C(t)] \left[y_D(S_{BP}(t)) + \theta_{DR} \sum_{j=1}^{N} a_{ij} R_j(t)\right] + \varepsilon \mu_i^C(t), \\
\dfrac{d\mu_i^B(t)}{dt} = -w_B - \delta_B \mu_i^A(t) + [\delta_B + h_B(S_{BQ}(t))] \mu_i^B(t) + \varepsilon \mu_i^C(t) + M_4 \left[y_B(S_{BP}(t)) + \theta_{BR} \sum_{j=1}^{N} a_{ij} R_j(t)\right] + \eta_{RB} \sum_{j=1}^{N} a_{ij} R_j(t) M_5 + \eta_{DB} \sum_{j=1}^{N} a_{ij} D_j(t) M_6, \\
\dfrac{d\mu_i^C(t)}{dt} = -\delta_R \mu_i^A(t) + (\varepsilon + \delta_R) \mu_i^C(t) - M_4 \left[p_R(S_A(t)) + \eta_{RB} \sum_{j=1}^{N} a_{ij} B_j(t)\right] + \theta_{DR} \sum_{j=1}^{N} a_{ij} D_j(t) [\mu_j^A(t) - \mu_j^C(t)] - \theta_{BR} \sum_{j=1}^{N} a_{ij} B_j(t) M_5, \\
0 \le t \le T, 1 \le i \le N, \lambda_i^B(t) - \lambda_i^C(t) = M_1, \lambda_j^C(t) - \lambda_j^B(t) = M_2, \lambda_j^A(t) - \lambda_j^B(t) = M_3, \mu_i^B(t) - \mu_i^C(t) = M_4, \mu_j^C(t) - \mu_j^B(t) = M_5, \mu_j^A(t) - \mu_j^B(t) = M_6.
\end{cases} \tag{13}
$$

Algorithm 1 Nash equilibrium computation strategy

Input: $\Bbbk = (G, \overline{S}_A, \overline{S}_{BP}, \overline{S}_{BQ}, \eta, \theta, p, y, h, \delta, \varepsilon, w, T, \mathbf{E}_0)$, error ς, maximum number of iterations K.
Output: Nash equilibrium (S_A^*, S_B^*).

$S_A^{(0)} = 0$; $S_B^{(0)} = 0$; $k = 0$;
repeat
  $k = k + 1$;
  take advantage of the system (3), $S_A = {S_A}^{(k-1)}$, $S_B = {S_B}^{(k-1)}$ and $\mathbf{E}(0)$ to forward calculate $\mathbf{E}(t)$;
  $\mathbf{E}^{(k)} := \mathbf{E}$;
  take advantage of the systems (13) with $S_A = {S_A}^{(k-1)}$, $S_B = {S_B}^{(k-1)}$, $\mathbf{E}^{(k)} := \mathbf{E}$, $\lambda(T) = \mu(T) = 0$, calculate λ and μ;
  $\lambda^{(k)} := \lambda$; $\mu^{(k)} := \mu$;
  take advantage of the system (15), (18), (19), $\mathbf{E} = \mathbf{E}^{(k)}$, $\lambda^{(k)} = \lambda$, $\mu^{(k)} = \mu$, calculate S_A, S_B;
  ${S_A}^{(k)} := S_A$, ${S_B}^{(k)} := S_B$;
until $\left\|{S_A}^{(k)} - {S_A}^{(k-1)}\right\| + \left\|{S_B}^{(k)} - {S_B}^{(k-1)}\right\| \leq \varsigma$ or $k > K$;
return $({S_A}^{(k)}, {S_B}^{(k)})$.

TABLE I: Summary information on the experimental datasets

Datasets	Nodes	Edges	Sources
Facebook	4039	88234	https://snap.stanford.edu/data/ego-Facebook.html
Twitter	81306	1768149	https://snap.stanford.edu/data/ego-Twitter.html
YouTube	495957	1936748	https://networkrepository.com/soc-youtube.php

refinements of the strategy combination. In each iteration, we utilize the model from (3) to perform the forward computation and obtain the evolution of user states. Next, we employ the system from (13) for the backward computation to acquire the associated adjoint functions, and finally compute the new strategy combinations via the systems in Eqs. (15), (18), (19). The entire iterative process monitors the cost budget associated with the rumor-mongering and debunking strategies. The process concludes when either the strategy combinations in two consecutive iterations are very close or the iteration limit is reached, outputting the optimized strategy combination.

VI. Experiments

In this section, we present comprehensive experiments on multiple datasets to validate the proposed method. Specifically, based on the concept of Nash equilibrium, we compare the gain of the rumor-monger and the loss of the debunker under different rumor-mongering and debunking strategies, and, in turn, examine the efficacy of the proposed strategy pair. We start the section, with a description of the experimental setup. Next, we analyze the variations in the obtained strategy pairs across three OSNs. Finally, we empirically validate the cost-effectiveness of the proposed hybrid rumor-debunking strategy through multiple comparative experiments on real Facebook, Twitter, and YouTube datasets and two actual rumor events. Additionally, we conduct a sensitivity analysis to explore the impact of some key parameters.

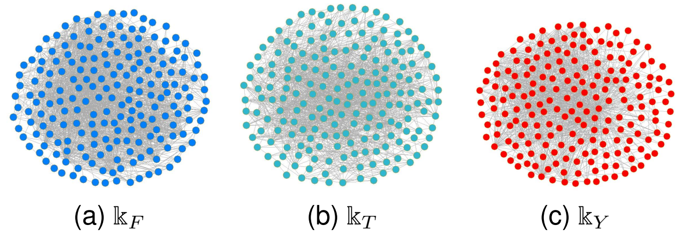


Fig. 4. Visualization of the 200-node subnets: (a) $\Bbbk_F$ of the Facebook network, (b) $\Bbbk_T$ of the Twitter network, and (c) $\Bbbk_Y$ of the YouTube network.

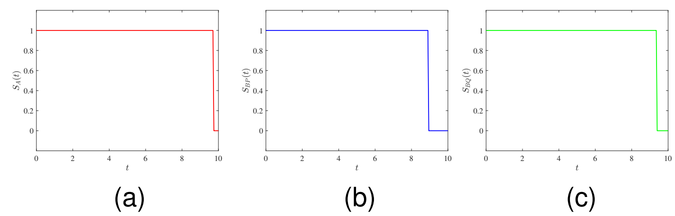


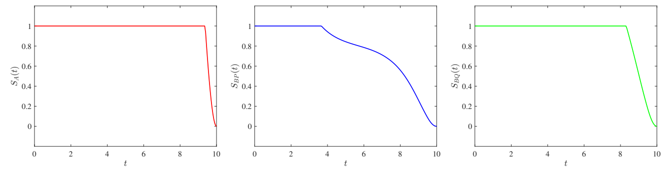
Fig. 5. Nash equilibrium pair for the Facebook example: (a) rumor-mongering strategy $S_A(t)$, (b) and (c) rumor- debunking strategies, $S_{BP}(t)$ and $S_{BQ}(t)$, respectively.

A. Experimental setup

All experiments are conducted using MATLAB R2022a. Standard, publicly available datasets are utilized for the analyses, including the Facebook [36], Twitter [36] and YouTube [37] datasets. The Facebook [36] dataset was collected using a dedicated Facebook app that was made available to surveyed participants. The (anonymized) data included in the dataset consists of various user-profile features (e.g., hometowns, birthdays, colleagues, etc.) and identified social circles that jointly define a network with 4039 nodes (users) and 88.234 edges (social connections). Similarly to the Facebook dataset, the Twitter [36] dataset also defines an online social network, which, in this case, consists of 81.306 nodes (users) and $1.768.149$ edges (social connections). The network is defined based on scrapped Twitter data consisting of hashtags and mentions, as described in detail in [36]. The last, the YouTube [37] dataset, defines a network of YouTube users and their relationships. A total of 495.957 nodes is used to model users, and $1.936.748$ edges are utilized to define the social relationships between the captured users. A summary of the datasets, including the number of nodes, connected edges and URLs, from which the three OSNs are available are listed in Table I. The selected datasets are used for simulation experiments with real social network structures with the goal of validating the effectiveness of the proposed method.

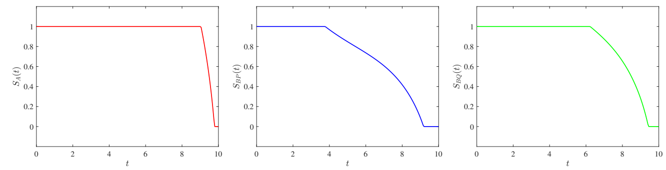
Given the massive scale of the original datasets, for the sake of feasibility, we conduct all experiments on subnetworks of the three original networks, denoted as $\Bbbk_F$, $\Bbbk_T$, and $\Bbbk_Y$. Making use of Pajek[1], we generate network graphs for these three subnetworks and show them in Fig. 4.

[1]http://mrvar.fdv.uni-lj.si/pajek/



(a) (b) (c)

Fig. 6. Nash equilibrium pair for the Twitter example: (a) rumor-mongering strategy $S_A(t)$, (b) and (c) rumor- debunking strategies, $S_{BP}(t)$ and $S_{BQ}(t)$, respectively.



(a) (b) (c)

Fig. 7. Nash equilibrium pair for the YouTube example: (a) rumor-mongering strategy $S_A(t)$, (b) and (c) rumor- debunking strategies, $S_{BP}(t)$ and $S_{BQ}(t)$, respectively.

B. Numeric examples

All experiments are conducted under the conditions detailed below. In the presented three examples, the initial network state is set to $\mathbf{E}_0 = (0.8, \cdots, 0.8, 0.1, \cdots, 0.1, 0.1, \cdots 0.1)$, $\overline{S}_A = \overline{S}_{BP} = \overline{S}_{BQ} = 1$, $T = 10$, whereas the remaining experimental parameters are determined based on network characteristics. It is important to note at this point that specific parameter values may vary under different circumstances. Due to the lack of actual data, certain parameter values are, therefore, chosen based on historical data estimates and assumptions, similarly to [9], [11].

Facebook example: For the rumor-mongering and rumor-debunking differential game problem (10) in the Facebook network:

$$\Bbbk_F = (G, \overline{S}_A, \overline{S}_{BP}, \overline{S}_{BQ}, \eta, \theta, p, y, h, \delta, \varepsilon, w, T, \mathbf{E}_0),$$

we set $\eta_{DB} = 0.19$, $\eta_{RB} = 0.15$, $\theta_{DR} = 0.13$, $\theta_{BR} = 0.17$, $\delta_R = 0.1$, $\delta_B = 0.1$, $\varepsilon = 0.12$, $(w_A, w_B) = (0.2, 0.2)$, $p(x) = (p_D, p_R) = (0.3x, 0.15x)$, $y(x) = (y_D, y_B) = (0.3x, 0.15x)$, and $h(x) = 0.22x$.

Twitter example: For the rumor-mongering and rumor-debunking differential game problem (10) in the Twitter network:

$$\Bbbk_T = (G, \overline{S}_A, \overline{S}_{BP}, \overline{S}_{BQ}, \eta, \theta, p, y, h, \delta, \varepsilon, w, T, \mathbf{E}_0),$$

we set $\eta_{DB} = 0.18$, $\eta_{RB} = 0.16$, $\theta_{DR} = 0.14$, $\theta_{BR} = 0.18$, $\delta_R = 0.1$, $\delta_B = 0.1$, $\varepsilon = 0.12$, $(w_A, w_B) = (0.2, 0.2)$, $p(x) = (p_D, p_R) = (0.3\sqrt{x}, 0.15\sqrt{x})$, $y(x) = (y_D, y_B) = (0.3\sqrt{x}, 0.15\sqrt{x})$, and $h(x) = 0.22\sqrt{x}$.

YouTube example: For the rumor-mongering and rumor-debunking differential game problem (10) in the YouTube network:

$$\Bbbk_Y = (G, \overline{S}_A, \overline{S}_{BP}, \overline{S}_{BQ}, \eta, \theta, p, y, h, \delta, \varepsilon, w, T, \mathbf{E}_0),$$

we set $\eta_{DB} = 0.17$, $\eta_{RB} = 0.16$, $\theta_{DR} = 0.17$, $\theta_{BR} = 0.18$, $\delta_R = 0.1$, $\delta_B = 0.1$, $\varepsilon = 0.12$, $(w_A, w_B) = (0.2, 0.2)$, $p(x) = (p_D, p_R) = \left(\frac{0.3x}{1+x}, \frac{0.15x}{1+x}\right)$, $y(x) = (y_D, y_B) = \left(\frac{0.3x}{1+x}, \frac{0.15x}{1+x}\right)$, and $h(x) = \frac{0.22x}{1+x}$.

Experiment 1: The objective of the rumor-mongering and rumor-debunking differential game problem is to identify Nash equilibrium strategy pairs. To determine the Nash equilibrium, we apply Algorithm 1 to the parameter settings of the Facebook, Twitter and Youtube examples, and report the results in Fig. 5, Fig. 6, and Fig. 7, respectively.

In the presented figures, (a) represents the rumor-mongering strategy $S_A(t)$, whereas (b) and (c) represent the rumor-debunking strategies, $S_{BP}(t)$ and $S_{BQ}(t)$, respectively.

Several interesting observations can be made from these results: (i) both the rumor-mongering strategy and the rumor-debunking strategy (gradually) decrease from the maximum to zero over time. This can be attributed to the two parties in the differential game of rumor-mongering and debunking ultimately reaching a Nash equilibrium. (ii) In the studied three examples, the time it takes for the Nash equilibrium to decrease varies due to differences in the network structures of the OSNs. (iii) Notably, for (b) and (c), the moment in time, at which the loss associated with the debunking strategies starts to decline, is not consistent across strategies. Therefore, distinct strategies can be devised for truth dissemination and regulatory measures, each aimed at minimizing the cost associated with mitigating the impact of rumors.

C. Basic strategy comparison validation

We validate the effectiveness of the proposed approach through multiple comparative experiments, including a random strategy, a uniform strategy, and the uncertainty of the rumor-mongering strategy. All experiments are conducted under the parameter settings described for the Facebook, Twitter, and YouTube examples.

1) Comparative experiment with the random strategy: The random strategy refers to a system that randomly allocates cost resources for rumor-mongering and rumor-debunking at each control time step. Because the control strategy adopted at each time step is random, both the rumor-monger and rumor-debunker randomly select the strategy to use. We devise an algorithm to generate a random strategy pair, as shown in Algorithm 2, and set $n = 100$, $h = 0.05$.

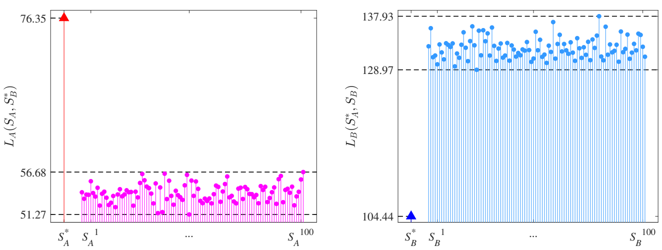
Experiment 2: Algorithm 2 is executed 100 times each under the parameter settings detailed with the definitions of the Facebook, Twitter and YouTube examples. Specifically, 100 rumor-mongering and rumor-debunking strategies are randomly generated within the upper and lower bounds of S_A, S_{BP} and S_{BQ}, denoted as $\mathbb{N}_A = \left\{S_A^1, \cdots, S_A^{100}\right\}$ and $\mathbb{N}_B = \left\{S_B^1, \cdots, S_B^{100}\right\}$ respectively. The net gain and total loss corresponding to each strategy are calculated and the generated results are shown in Figs. 8, 9, and 10.

Figs. 8a, 9a and 10a illustrate $L_A(S_A, S_B^*)$ for the three studied examples, where $S_A \in \{S_A^*\} \cup \mathbb{N}_A$. It is easy to see that $L_A(S_A^*, S_B^*) > L_A(S_A, S_B^*)$, $S_A \in \mathbb{N}_A$. Similarly, Figs.

Algorithm 2 Random strategy generation

Input: $\Bbbk = (G, \overline{S}_A, \overline{S}_{BP}, \overline{S}_{BQ}, \eta, \theta, p, y, h, \delta, \varepsilon, w, T, \mathbf{E}_0)$, integer n, and step size h.
Output: Random strategy pair (S_A, S_B).

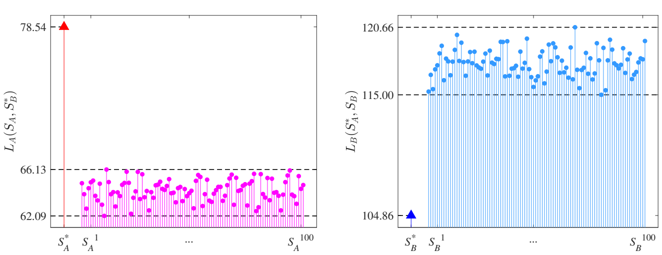
```
pick out n − 1 points within the interval [0, T] by step size h,
    denoted as t_k, k = 1, ..., n − 1, where 0 = t_0 < t_1 < ··· < t_{n−1} < t_n = T;
for 0 ≤ k ≤ n − 1 do
    randomize α ∈ [0, S̄_A), β ∈ [0, S̄_BP), γ ∈ [0, S̄_BQ];
    for 0 ≤ i ≤ m − 1 do
        t_k^i = t_k + (i/m) h;
        S_A(t) := α, S_BP(t) := β, S_BQ(t) := γ;
    end for
end for
S_A(t_n) := S_A(t_{n−1}), S_B(t_n) := S_B(t_{n−1});
return (S_A, S_B).
```



(a) (b)

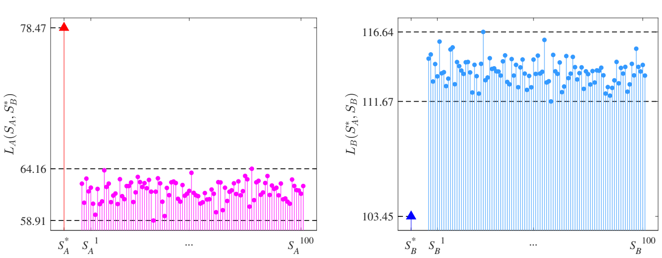
Fig. 8. Comparative results with the random strategy for the Facebook example: (a) $L_A(S_A, S_B^*)$, (b) $L_B(S_A^*, S_B)$.

8b, 9b, and 10b depict $L_B(S_A^*, S_B)$ for the three investigated examples, where $S_B \in \{S_B^*\} \cup \mathbb{N}_B$. Again, it can be concluded that $L_B(S_A^*, S_B^*) < L_B(S_A^*, S_B), S_B \in \mathbb{N}_B$. Overall, the results from Figs. 8–10, suggest that when Nash equilibrium is employed, the rumor-monger gains the most and the rumor-debunker loses the least.



(a) (b)

Fig. 9. Comparative results with the random strategy for the Twitter example: (a) $L_A(S_A, S_B^*)$, (b) $L_B(S_A^*, S_B)$.



(a) (b)

Fig. 10. Comparative results with the random strategy for the YouTube example: (a) $L_A(S_A, S_B^*)$, (b) $L_B(S_A^*, S_B)$.

Experiment 3: To further validate the effectiveness of the proposed approach in large-scale networks, we conduct experiments (using the Facebook network as an example) by increasing the number of network nodes. Specifically, we construct networks with 1000 and 2000 nodes, denoted as $\Bbbk_{F1000}$ and $\Bbbk_{F2000}$, respectively. Algorithm 2 is executed 100 times under the parameter settings used previously for the Facebook data. Consequently, 100 random rumor-mongering and rumor-debunking strategies are obtained, denoted as $\mathbb{N}_A = \{S_A^1, \cdots, S_A^{100}\}$ and $\mathbb{N}_B = \{S_B^1, \cdots, S_B^{100}\}$ respectively. The corresponding net gain and total loss of the Nash equilibrium strategy and each random strategy combination are illustrated in Fig. 11.

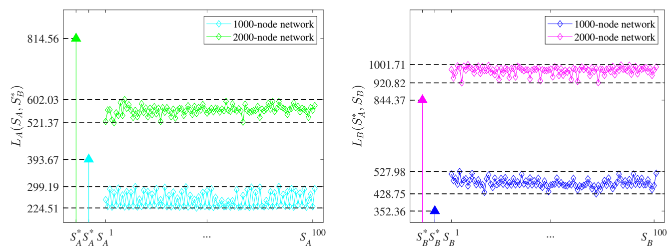


Fig. 11. Comparison result in $\Bbbk_{F1000}$ (left) and $\Bbbk_{F2000}$ (right).

From Fig. 11, it can be intuitively seen that with the increase in network scale, both the gain and loss also increase. When adopting the Nash equilibrium strategy, the rumor-monger maximizes gains while the rumor-debunker minimizes losses. The Nash equilibrium strategy represents the optimal choice for both parties. Thus, the effectiveness of the proposed method is demonstrated under large-scale networks.

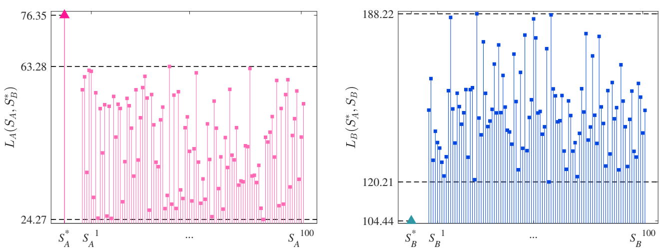
2) Comparative experiment with the uniform strategy: The so-called uniform strategy refers to a system that evenly distributes cost resources for rumor-mongering and rumor-debunking at each control time step. It entails that both the rumor-monger and rumor-debunker adopt the same strategy in the long run without any strategy changes. We design an algorithm for generating a uniform strategy pair, as shown in Algorithm 3, and set $n = 100$, $h = 0.05$.

Algorithm 3 Uniform strategy generation

Input: $\Bbbk = (G, \overline{S}_A, \overline{S}_{BP}, \overline{S}_{BQ}, \eta, \theta, p, y, h, \delta, \varepsilon, w, T, \mathbf{E}_0)$, a positive integer n, and a step size h.
Output: Uniform strategy pair (S_A, S_B).

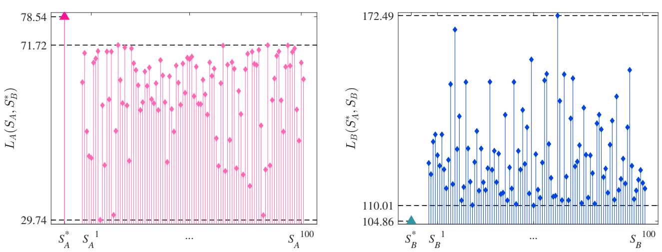
```
pick out n − 1 points within the interval [0, T] by step size h,
    denoted as t_k, k = 1, ..., n − 1, where 0 = t_0 < t_1 < ··· < t_{n−1} < t_n = T;
randomize α ∈ [0, S̄_A), β ∈ [0, S̄_BP), γ ∈ [0, S̄_BQ];
for 0 ≤ k ≤ n − 1 do
    for 0 ≤ i ≤ m − 1 do
        t_k^i = t_k + (i/m) h;
        S_A(t) := α, S_BP(t) := β, S_BQ(t) := γ;
    end for
end for
S_A(t_n) := S_A(t_{n−1}), S_B(t_n) := S_B(t_{n−1});
return (S_A, S_B).
```

Experiment 4: Algorithm 3 is executed 100 times each under the parameter settings discussed when introducing the Facebook, Twitter and YouTube examples. Specifically, 100 uniform rumor-mongering and rumor-debunking strategies are generated within the upper and lower bounds of S_A, S_{BP}, and S_{BQ}, denoted as $\mathbb{N}_A = \{S_A^1, \cdots, S_A^{100}\}$ and $\mathbb{N}_B = \{S_B^1, \cdots, S_B^{100}\}$ respectively. The net gain and total loss corresponding to each strategy are then calculated, and the generated results are shown in Figs. 12, 13, and 14.



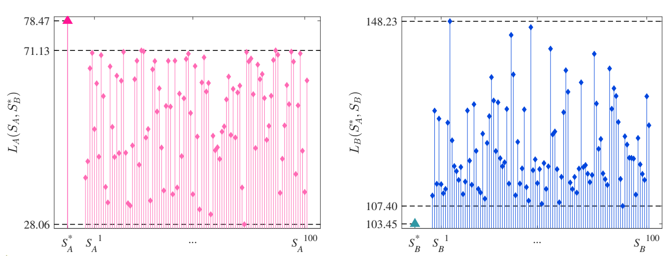
(a) (b)

Fig. 12. Comparative results with the uniform strategy in the Facebook example: (a) $L_A(S_A, S_B^*)$, (b) $L_B(S_A^*, S_B)$.



(a) (b)

Fig. 13. Comparative results with the uniform strategy in the Twitter example: (a) $L_A(S_A, S_B^*)$, (b) $L_B(S_A^*, S_B)$.



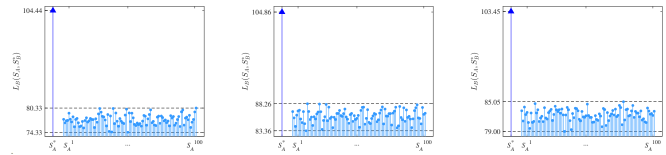
(a) (b)

Fig. 14. Comparative results with the uniform strategy in the YouTube example: (a) $L_A(S_A, S_B^*)$, (b) $L_B(S_A^*, S_B)$.

Figs. 12a, 13a and 14a show the value of $L_A(S_A, S_B^*)$ for the three studied examples, where $S_A \in \{S_A^*\} \cup \mathbb{N}_A$. From the results, it can be seen that $L_A(S_A^*, S_B^*) > L_A(S_A, S_B^*), S_A \in \mathbb{N}_A$. Similarly, Figs. 12b, 13b and 14b present $L_B(S_A^*, S_B)$ for our three examples, where $S_B \in \{S_B^*\} \cup \mathbb{N}_B$, and we again conclude that $L_B(S_A^*, S_B^*) < L_B(S_A^*, S_B), S_B \in \mathbb{N}_B$. From the reported results, we again observe that the rumor-monger gains the most and the rumor-debunker loses the least when the Nash equilibrium is employed.

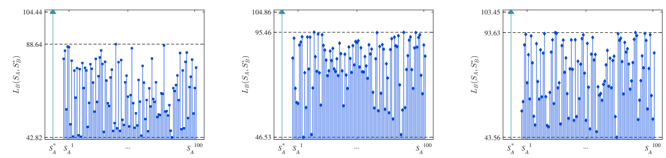
3) Comparative experiment with the uncertain rumor-mongering strategy: Due to the lack of information and limited expertise, the rumor-monger may not be able to accurately estimate the specific profit of $L_A(S_A, S_B)$. In this context, the rumor-monger loses the ability to confirm the values of (S_A^*, S_B^*)To evaluate the advantage of the rumor-debunking strategy S_B^*, we compare the loss of the rumor-debunker under random and uniform rumor-mongering strategies.

Experiment 5: In Experiments 2 and 4, we generated 100 random rumor-mongering strategies and 100 uniform rumor-mongering strategies, denoted as $\mathbb{N}_A = \{S_A^1, \cdots, S_A^{100}\}$ and then computed the total loss for each strategy.



(a) Facebook example (b) Twitter example (c) YouTube example

Fig. 15. Comparison with the uncertainty of random rumor-mongering strategy.



(a) Facebook example (b) Twitter example (c) YouTube example

Fig. 16. Comparison with the uncertainty of uniform rumor-mongering strategy.

Fig. 15 plots $L_B(S_A, S_B^*)$, $S_A \in \{S_A^*\} \cup \mathbb{N}_A$ of Experiment 2 for the three considered OSNs, and it can be seen that $L_B(S_A, S_B^*) < L_B(S_A^*, S_B^*), S_A \in \mathbb{N}_A$. Similarly, Fig. 16 plots $L_B(S_A, S_B^*)$, $S_A \in \{S_A^*\} \cup \mathbb{N}_A$ of Experiment 4 on the three OSNs, and we again observe that $L_B(S_A, S_B^*) < L_B(S_A^*, S_B^*), S_A \in \mathbb{N}_A$. It is interesting to note that in the case of uncertain rumor-mongering strategies, regardless of whether a random or uniform strategy is employed, the rumor-debunker loss is lower than with the Nash equilibrium. Therefore, we conclude that the overall loss of the rumor-debunker is always lower than $L_B(S_A^*, S_B^*)$ when the Nash strategy S_B^* is adopted. This indicates that S_B^* at the Nash equilibrium can effectively reduce the loss of the rumor-debunker.

D. Model comparison

Next, we verify the effectiveness of the proposed hybrid rumor-debunking strategy by comparing the overall loss of the rumor-debunker and the evolution of the B state in the network with competing models. Specifically, we compare against the work from [32], which focuses on rumor debunking by solely spreading the truth. To visually observe the rumor propagation in the network, we estimate $\overline{\mathbf{E}}(t) = \left(\overline{D}(t), \overline{B}(t), \overline{R}(t)\right)$, which represents the expected state evolution trajectory of the network, where:

$$\bar{D}(t) = \frac{1}{N}\sum_{i=1}^{N} D_i(t), \overline{B}(t) = \frac{1}{N}\sum_{i=1}^{N} B_i(t), \bar{R}(t) = \frac{1}{N}\sum_{i=1}^{N} R_i(t). \quad (22)$$

Experiment 6: Given that the functions $h(x), p(x)$ and $y(x)$ represent rumor-mongering and debunking strategies, we control these variables for model comparison. We conduct experiments in 1000-node Facebook, Twitter and YouTube networks with the same parameters as above. Specifically, we consider three distinct cases:

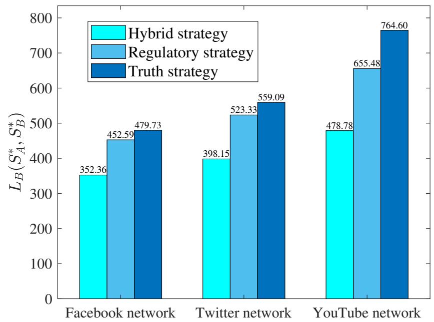


Fig. 17. Comparison of the model over the three networks.

TABLE II: Estimated parameters for two events.

Parameters	Dataset_R1	Dataset_R12	Parameters	Dataset_R1	Dataset_R12
η_{DB}	0.237	0.036	ε	0.001	0.001
η_{RB}	0.966	0.135	p_D	0.019	0.011
θ_{DR}	0.001	0.069	p_R	0.001	0.993
θ_{BR}	0.991	0.209	y_D	0.001	0.001
δ_R	0.263	0.001	y_B	0.154	0.151
δ_B	0.001	0.001	h_B	0.607	0.626

- Case 0 (Truth strategy): Only the truth dissemination strategy is implemented, where $h(x) = 0$, $p(x) \neq 0$ and $y(x) \neq 0$. This corresponds to the work from [32].
- Case 1 (Regulatory strategy) : Only the regulatory strategy is implemented, where $y(x) = 0$, $p(x) \neq 0$ and $h(x) \neq 0$.
- Case 2 (Hybrid strategy) : Two control strategies are implemented, where $p(x) \neq 0$, $y(x) \neq 0$, and $h(x) \neq 0$.

Fig. 17 illustrates $L_B(S_A^*, S_B^*)$ for the three studied examples. It is evident that across the three examples, the proposed model (marked hybrid) exhibits the lowest rumor-debunking loss. We hence conclude that, compared to the adoption of a single rumor-debunking strategy, the collaboration of two strategies results in the lowest rumor-debunking loss, which verifies the effectiveness of our hybrid debunking strategy.

Fig. 18 shows the dynamic evolution of $\overline{B}(t)$ under different strategies in three 1000-node networks. It can be observed that over time, the density of rumor-believing in the network first increases, then gradually decreases, and eventually stabilizes. It is evident that under the hybrid rumor-debunking strategy model, the probability of believing in rumors is the lowest, and the effect of rumor suppression is the best.

E. Validation with actual rumor events

Subsequently, we validate the effectiveness of the proposed propagation model using actual rumor events. Inspired by the work in [38], we initially estimate all parameters in the model using a portion of the data and then leverage the remaining data for model validation. To ensure that the proposed model can capture the propagation process associated with various rumors, we select two specific rumor events for validation.

The data used in this experiment originates from the Newly Emerged Rumors in Twitter (NERT) dataset [39], which empirically investigates the dissemination patterns of newly emerging rumors on Twitter. This extensive dataset comprises 12 distinct rumor events, each accompanied by the simultaneous spread of anti-rumors. After a thorough analysis, we select the events in the Dataset_R1 and Dataset_R12 for our experiments, as they offer larger scales, comprehensive information, and relatively stable fluctuations in rumor propagation processes, and are thus ideal for rigorous experimentation.

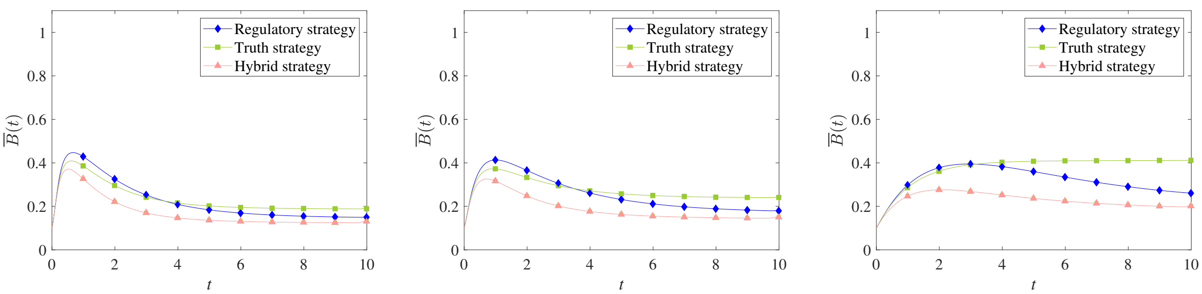
In the NERT dataset, each row represents a tweet related to a rumor, with each column providing information relevant to that tweet. Specifically, the status column marked “r” represents rumor tweets, corresponding to the B state in the proposed model, while “a” represents anti-rumor tweets, corresponding to the R state. We calculate the hourly rumor (anti-rumor) propagation density in the network by dividing the number of rumor (anti-rumor) tweets within each hour by the total number of rumor (anti-rumor) tweets. Notably, since the original dataset does not contain network structure information, as the rumor events are captured from Twitter, we utilize a dataset [36] to construct a real Twitter network for simulating the rumor propagation process within our model.

For the events Dataset_R1 and Dataset_R12, rumor fluctuations lasted for 49 hours and 77 hours, respectively. When choosing data for experimental fitting, we must strike a balance: selecting too little data may not yield enough information, while an excessive amount, especially after the rumor trend has stabilized, may not be representative. Therefore, we opt to use approximately the first 20% of each event’s duration for parameter estimation, specifically, the first 9 hours of Dataset_R1 and the first 17 hours of Dataset_R12. To identify the best model parameters, we employ sequential quadratic programming, continuously fine-tuning all parameters within the 0.001 to 0.999 range until the sum of squared errors is minimized. This process yields parameter estimates for both events, which are listed in Table II. Substituting these parameters into our proposed model allows us to generate predicted density curves for both $\bar{R}(t)$ and $\bar{B}(t)$.

Figs. 19 and 20 compare our model’s predictions with the actual density variations observed in two actual rumor events. Clearly, the model’s predictions closely match the real-world trends of rumor propagation and debunking in both cases. This underscores the effectiveness of the proposed model in fitting real-world diffusion data accurately.

F. Sensitivity analysis

Finally, we explore the impact of key parameters on the cost-effectiveness of the Nash-equilibrium strategy. By controlling for the value of selected variables, a systematic investigation is conducted into how the expected net gain $L_A(S_A^*, S_B^*)$ for the rumor-monger and the total loss $L_B(S_A^*, S_B^*)$ for the debunker vary with different parameter values under the Nash-equilibrium strategy. Specifically, the effects of the rumor-debunking control cost budget $\overline{S}_B = (\overline{S}_{BP}, \overline{S}_{BQ})$ and the average rumor gain/loss $w = (w_A, w_B)$ on the cost-effectiveness of the Nash equilibrium strategy are investigated. Three example models, denoted as $\mathbb{k} \in \{\mathbb{k}_F^{1000}, \mathbb{k}_T^{1000}, \mathbb{k}_Y^{1000}\}$, are considered in three social networks with 1000 nodes each. By setting $\overline{S}_B = (\overline{S}_{BP}, \overline{S}_{BQ}) \in$



(a) Facebook example (b) Twitter example (c) YouTube example

Fig. 18. Comparison of the dynamic evolution of $\overline{B}(t)$ under different models in three networks.

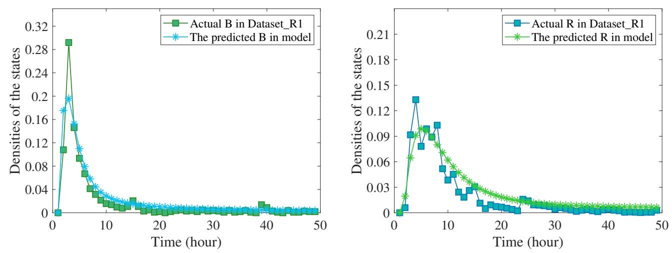


Fig. 19. Comparison of model prediction with Dataset_R1.

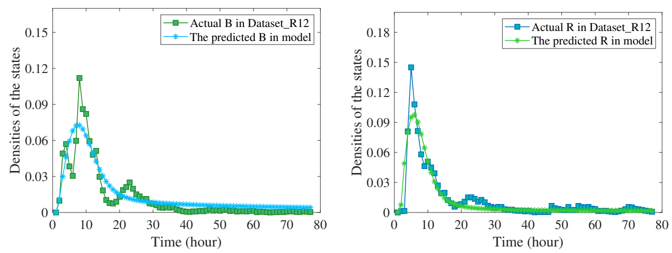


Fig. 20. Comparison of model prediction with Dataset_R12.

$\{0.2, 0.4, \cdots 1.8\}$, with other parameters remaining consistent with the aforementioned examples. Algorithm 1 is executed to obtain the Nash strategy combinations and calculate the total gain for the rumor-monger and the total loss for the debunker, as illustrated in Fig. 21. For $w = \{0.1, 0.2, \cdots 0.5\}$, with other parameters held constant, Algorithm 1 is again executed, and the results are displayed in Fig. 22.

Fig. 21 illustrates that, regardless of network structure, as the debunking control cost budget increases, both the rumor-monger's gain and the debunking loss decrease. However, after the rumor-debunking control cost budget reaches a certain level, this downward trend gradually flattens out. This suggests that appropriately increasing the debunking control budget helps suppress rumors and reduce associated losses. Fig. 22 shows that, across different social networks, as the average rumor gain/loss increases, both the rumor-monger's gain and the debunking loss increase. This indicates that larger rumor gains and losses are more detrimental to the rumor debunker. Therefore, it is crucial to strengthen the regulation of rumors and minimize their potential gains in order to achieve effective rumor control.

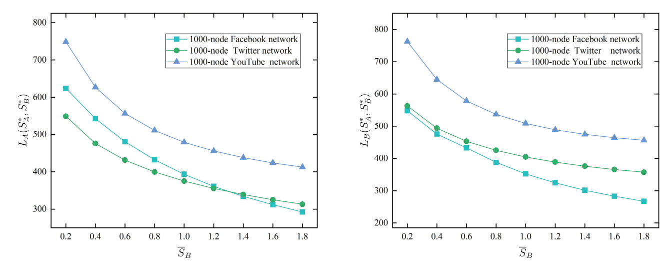


Fig. 21. Impact of rumor-debunking control cost budget on cost-effectiveness.

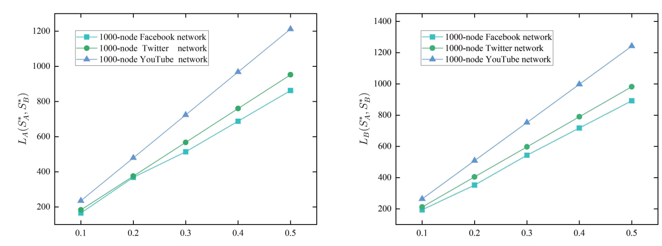


Fig. 22. Impact of average rumor gain/loss on cost-effectiveness.

VII. Discussions

This paper offers new perspectives for in-depth research on online rumors and provides feasible theoretical support for the design of rumor control strategies, thus possessing significant practical meaning and application value. Specifically, when unverified rumors emerge on the network, the platform can promptly initiate a regulatory mechanism and classify users based on user profiles and social relationships. For users who frequently disseminate information and have extensive network connections, authoritative rumor-debunking content should be preferentially pushed to them, leveraging their social influence to accelerate the spread of the truth. As for malicious users, measures such as blocking and restricting the forwarding and viewing of rumors can be taken. Through such targeted measures, the spread of rumors can be effectively curbed, and public opinion can be guided in a positive direction. Currently, major social platforms have begun to implement fact-checking and content review mechanisms in different regions. For example, Weibo has set up a dedicated rumor-refuting account to promptly release authoritative information for clarification. Facebook has launched a fact-checking mechanism and cooperated with third-party institutions to mitigate adverse effects by labeling and reducing the spread of rumors.

There are some issues that remain to be discussed. Our network dataset is derived from mainstream social platforms, which limits its coverage of niche social networks or platforms specific to certain professional domains. This may hinder the results from fully representing a broader and more diverse social network environment. Additionally, the rumor event dataset is sourced from specific social platforms and time periods, resulting in insufficient sample representativeness, which could impact the generalizability of the experimental findings. Given the diversity of social platforms and cultural contexts, data from different platforms or cultural backgrounds may exhibit varying network structures and propagation characteristics. To address dataset limitations, future research will incorporate more diverse data sources, including comprehensive datasets that span across platforms, cultures, and regions, to further enhance applicability.

VIII. Conclusions and Future Work

In this paper, we addressed the issue of hybrid debunking strategies in the context of adversarial behaviors between rumor-mongering and debunking. We proposed a novel node-based dynamical model to describe the spread of rumors on arbitrary networks and utilized differential game theory to characterize the processes of rumor-mongering and debunking. We conducted extensive comparative experiments on three real OSNs, including comparisons with random strategy, uniform strategy, and single strategy models to evaluate the effectiveness of the proposed hybrid debunking approach. Additionally, we utilized two actual rumor events for parameter estimation and prediction to further validate the efficacy of our propagation model. The results demonstrate that the proposed model effectively simulates the propagation trends of rumor and debunking in real social networks. Finally, we conducted a sensitivity analysis of the parameters, offering valuable insights for effectively controlling the propagation of rumors within networks.

While the differential game framework provides a solid theoretical foundation, its practical implementation in large, dynamic OSNs remains a significant challenge. The real-time calculation of optimal debunking strategies across vast and constantly changing networks introduces substantial computational complexity. It is essential to address the practical feasibility of deploying these strategies in real-world scenarios, particularly concerning computational costs and the time-sensitive nature of rumor control. For computational costs, we plan to adopt two methods: one is to optimize algorithms, such as using approximate algorithms or heuristic algorithms to seek suboptimal solutions, which can significantly improve computational efficiency while ensuring a certain level of accuracy; The second is to simplify the model by reducing its complexity based on reasonable assumptions and approximation methods, making it easier to handle. Concerning the time-sensitive aspect, we will strive to achieve the precise classification of user groups for rapid deployment of strategies. Specifically, for users experiencing frequent changes in network structures, we will allocate additional computational resources to ensure efficient computation of game strategies. Conversely, for users with relatively stable network structures, resource allocation will be moderately reduced. This approach allows for a comprehensive understanding and effective utilization of the characteristics of social users, while also enabling flexible responses to dynamic changes in network structures, thereby alleviating resource constraints and maximizing the potential of computing resources.

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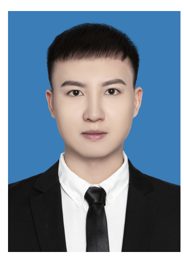

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